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On convection under an air surface

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A shallow layer of silicone oil on a plane, circular copper plate was uniformly heated from below. The air on its surface was kept at constant temperature by a uniformly cooled glass plate close to the oil, which at the same time inhibited air motions. Motions began with concentric circular rolls which, after the centre ring had formed, broke down into a hexagonal pattern. A rather accurate determination of the wavelength of the motions was possible. The wavelength was found to be variable with the depth of the fluid layer in qualitative accordance with the theory of Nield. Supercritical motions are briefly discussed.

1. Introduction

As is well known, experiments on convection in a shallow layer of fluid heated from below and cooled by an air surface produce a pattern of hexagonal cells, when conditions are sufficiently uniform in the horizontal. The most regular patterns ever observed seem to be those in Bénard's (1900) original paper. Following experimental evidence obtained by Block (1956), it has been shown theoretically by Pearson (1958), Scriven & Sternling (1964), Nield (1964), and Smith (1966) that surface tension was most likely an essential factor in Bénard's experiments. It has, furthermore, been shown experimentally by Koschmieder (1966) that under a rigid lid convective motions do not exhibit a hexagonal pattern, but a pattern of rolls, whose orientation is determined by the shape of the lateral wall. Some pictures which show the establishment of a hexagonal pattern under an air surface have been published recently by Segel (1966). These pictures are unsatisfactory insofar as there is no control of the air above the oil layer. For a comparison with theory the air covering the fluid layer, should have as uniform a temperature as possible and at the same time the air motions should be suppressed as far as possible. Since a surprisingly simple way to achieve both objectives appeared, the subsequently described study was undertaken.

2. Description of the apparatus

The apparatus used is a slightly improved version of a set-up used previously by Koschmieder. A 12 mm thick copper plate of 200 mm diameter served as bottom and was heated from below by a brass disk with ten concentric resistance wires in series. The copper plate and the brass plate are separated by a shallow water tank in which a stirrer rotated to smooth the temperature distribution in the brass plate. The lateral wall of the oil layer was a 1 mm thick lucite ring from which, 7·5 mm above the copper plate, a shoulder extended horizontally outwards, on which the lid rested. The lucite ring was surrounded by 50 mm foam rubber for thermal insulation. The bottom of the lid was a 1·9 mm thick glass plate of 216 mm diameter on to which a 60 mm high Bakelite wall was attached. The glass plate was cooled by a water flow of around 200 cm^3/sec from a controlled bath. The water jet impinges on the centre of the plate and is then forced to spread evenly to the sides under a lucite disk. This arrangement provides a good approximation to a uniformly cooled lid. The lid and the oil on the copper plate are separated by an air layer which is sealed in at the circle where the glass plate rests on the shoulder of the rim. The minimum distance from the oil to the glass was determined by the need to avoid contact of the oil, whose level was raised at the rim due to the adhesion of the oil to the lucite ring. This closest distance was 0·75 mm. With such a narrow gap, the air upon the oil virtually stayed at rest and obtained a reasonable uniform temperature from the glass plate. The silicone oil was Dow Corning 200 fluid of 1 stoke viscosity, of density $\rho = 0{\cdot}968\,\mathrm{g\,cm^{-3}}$, expansion coefficient $\alpha = 0{\cdot}96 \times 10^{-3}\,{}^{\circ}\mathrm{C}^{-1}$, specific heat $C_p = 0{\cdot}35\,\mathrm{cal\,g^{-1}\,{}^{\circ}C^{-1}}$, thermal conductivity $k = 0{\cdot}37 \times 10^{-3}\,\mathrm{cal\,cm^{-1}\,sec^{-1}\,{}^{\circ}C^{-1}}$ and surface tension $S = 20{\cdot}9\,\mathrm{dyn\,cm^{-1}}$ at 25 °C. The dependence of surface tension on temperature was measured and found to be $-\sigma_0 = 0{\cdot}058\,\mathrm{dyn\,cm^{-1}\,{}^{\circ}C^{-1}} \pm 5\,\%$. The overall temperature difference between the cooling water and the bottom plate was measured with copper-constantan thermocouples ($1\,\mu\mathrm{V} = 2{\cdot}5 \times 10^{-2}\,{}^{\circ}\mathrm{C}$). The motions of the oil were made visible with aluminum powder. Dark lines in the pictures mean vertical motions, while areas of predominantly horizontal motions appear bright. Pictures were taken with the lid removed for about 20 sec. The copper bottom plate was not blackened.

3. The onset of convection

The establishment of a hexagonal pattern under an air surface is shown in figures 1–6, plates 1–3. The carefully levelled copper plate was in this case covered by a $4{\cdot}32 \pm 0{\cdot}02$ mm deep oil layer, which was gradually heated starting from zero vertical temperature difference. Rings formed consecutively inward from the rim. Four hours after heating started there were eight rings at 370 μV temperature difference. If the vertical temperature difference was steadied out, the rings have been observed unchanged for hours. Twelve rings could be observed at 380 μV 15 min later. Five more minutes and fifteen concentric rings had just established (see figure 1). The lid had not been lifted so far and the motions were still almost axially symmetric. The motions had developed precisely the same way as with the lid in contact with the fluid, but the horizontal dimensions of the rings differed considerably. While in the latter case the concentric rings stay up to five times the critical temperature difference, under an air surface the rings break down into hexagons immediately. The beginning of the breakdown of axial symmetry can be seen in figure 2, taken 8 min after figure 1, at a temperature difference of 390 μV. The conversion into hexagons proceeds in a well-defined sequence. The originally smooth rolls become bulgy in a very well co-ordinated way for a pair of rolls with ascending motions in the middle. Where the bulges of the undulating rolls meet at the centreline of the pair of rolls they narrow rapidly to form a 'pinch'. Top view pictures as in Segel's article are very revealing. Where the pinches are, the rolls are soon disconnected by fine, transparent, straight lines, perpendicular to the original axis of the rolls. The cells, which thus have established themselves, slowly adopt a more circular form and finally arrange to fill the available space with proper hexagons. Figure 3 shows how the first cells have already become established in the outer rings, while the centre rings begin to decay. In figure 4, the hexagonal pattern is established, although some irregularities remain. The two rolls adjacent to the rim stay virtually unaffected. Almost an hour later at the same temperature difference (figure 5) the cells have a little more regular boundaries. There is one hexagon right at the centre of the plate and then in rings consecutively from the centre there are 6–12–18–24–29–33 not always perfect cells. The outermost cell ring never managed to align perfectly to the cells further inwards. Finally figure 6 shows, at considerably increased temperature difference, the motions still in a hexagonal pattern. The number of cells in the cell rings are 1–6–12–18–23–31–35. Both rolls adjacent to the rim are now cut into cells shaped like pentagons, to accommodate to the shape of the rim. The falling out of aluminum powder has accumulated tiny heaps at the centres of the cells, thereby possibly stabilizing the pattern. The cells themselves have become very regular, if the available space permitted.

4. Determination of the wavelength

The appearance of the concentric rings prior to the breakdown into hexagons permits an accurate determination of the wavelength of the convective motion. The knowledge of the wavelength again is essential to check the theory of Nield, which matches precisely the experimental situation here. Nield studies convection due to buoyancy *and* surface tension on a good heat-conducting plate, say copper as here, and under an upper medium of variable heat conductivity, ranging from excellent conduction to plain insulation. His result is, that the wavelength should vary accordingly from longer waves at bad conduction to shorter ones at excellent conduction.

At first a series of experiments has been made to determine the wavelength of the motions in the case where thirteen concentric rolls occur with exactly the same setup as described above. From sixteen consecutive runs which produced thirteen rings the depth d for thirteen rings was determined to be $d = 5{\cdot}145 \pm 0{\cdot}04$ mm (standard deviation). These experiments covered all possible diameters of the centre ring which consequently produce different arrangements of the cells in the cell rings. The average d coincided with the depth for the most regular distribution of the cells in the different rings, in other words with harmonic rings on the whole plate. From the knowledge of d it follows that the wavelength for *one* roll $\lambda = \pi/a_c = r/nd = 1{\cdot}495 \pm 0{\cdot}012$ for thirteen rings (a_c is the critical wave-number). The magnitude of the error ($\pm 0{\cdot}8$ %) is not surprising if one keeps in mind that the largest possible error here is one-half wavelength, corresponding to ± 4 % for thirteen rings.

It should be noted, that the measured λ deviates considerably from the value 1·17 obtained from Rayleigh's theory for a free upper surface by Reid & Harris (1958). It differs even more from the value 1·008 for rigid, rigid boundaries, which value was confirmed by Koschmieder's experiment with an accuracy of better than 3 %. But the λ we have measured here is compatible with Nield's values, which range up to 1·57 for the case of insulation at the upper surface. It is difficult to assess how effective the insulation on top of the oil is, since Nield expresses the heat conductivity in terms of a dimensionless parameter $L = q_0 d/k_c$. The quantity q_0 is the rate of change with temperature of the heat flux from the fluid surface, d is the depth of the fluid and k_c its heat conductivity. We know nothing about the value of q_0. Since for thirteen rings, L is of order 0·4 according to the measured λ ($L = 0$ is perfect insulation), and since k_c^{-1} is of order 10^3, q_0 has to be very small here. The most conveniently variable quantity in L is the depth. The wavelengths have therefore been determined for several d's, with corresponding variations in the thickness of the air layer, since the lid position could not easily be

rings	depth (mm)	λ	$\Delta T(\mu V)$	ΔT_c(theor.) (μV)	R_c	L
11	6·76	1·347	140	120	790	1·3
13	5·15 ± 0·04	1·50 ± 0·01	251 ± 5	243	710	0·4
15	4·28 ± 0·04	1·56 ± 0·016	385 ± 9	409	682	0·1

TABLE 1

changed. The average depth for fifteen rings (figures 1–5) was found to be 4·28 ± 0·04 mm, corresponding to $\lambda = 1{\cdot}56$. That means the wavelength increased 4 % with decrease of d. The measured λ came now very close to the maximal possible in Nield's theory, namely $\lambda_{\max} = 1{\cdot}576$. Attempts to produce seventeen concentric rings at $d = 3{\cdot}8$ mm failed. Instead of the innermost rings there appeared clusters of hexagonal cells, indicating that near the centre the experimental conditions were no longer sufficiently controlled. The wavelength inferred from the final hexagonal pattern scattered and therefore these experiments were discontinued. On the other hand eleven harmonic rings appeared at 6·76 mm, corresponding to $\lambda = 1{\cdot}35$, a decrease of 8 % due to the increased depth. The air layer was now 0·75 mm thick, which was the narrowest ever tried. Table 1 summarizes the results of these experiments.

The measurements of the overall temperature difference ΔT should be considered as qualitative, since the temperature loss in the air layer is involved. Assuming the same heat flux through the air as through the silicone layer, and the thermal conductivities of the resting substances as valid, the temperature loss in the air layer will be greater than that in the oil, if the depth of the air layer is greater than $\frac{1}{6}d$. Under the same assumptions convection in the air layer should set in, if the depth of the air layer is approximately $1{\cdot}2d$. The experiment with seventeen rings is close to that condition. Actually it can never be assumed that the air is resting even in the smallest air gaps. There may be motions due to a small radial temperature gradient or motions, which match the motions in the oil layer. But no information, concerning these motions is available. Yet the measurements of ΔT can be taken as a qualitative proof that the critical Rayleigh number in these experiments is smaller than the value 1100 which follows from Rayleigh's theory for a free upper surface. The values of R_c which Nield obtains are listed in table 1 for comparison. The calculated ΔT_c are based on these values. Finally, the approximate values of L according to the measured λ are listed. One should keep in mind that excellent conduction on the upper surface means at least $L = 1000$. Nield extends his calculations to $L = 10^{10}$. The critical Marangoni number $B_c = \sigma_0 \Delta T d/\rho\nu\kappa$ for $d = 6{\cdot}76$ mm and $L = 1{\cdot}3$ is 126 according to Nield. The value we obtain from this experiment is $B_c = 85$, if we assume resting air on the fluid. Assuming no temperature loss across the air layer, one obtains $B_c = 138$.

In a number of experiments it has been tried to influence λ by changing the value of q_0. For this purpose a ground $\frac{1}{2}$ inch thick brass plate has been placed at different distances from the oil level, which was held constant at 5·15 mm. It turned out that the brass lid was too heavy to be carried by the 1 mm lucite rim, therefore the lid was placed on lucite rings of 5 mm thickness, 190 mm inner diameter and different heights, inserted into the otherwise unchanged set-up. The brass extended 2 mm down from the level of the supporting rings, except for an area 3 mm wide at the ring to stay away from the oil level raised by the adhesion. In this way the brass could be brought as close as 0·5 mm to the oil. A radius of 95 mm corresponds to 12·35 times the horizontal extent of one roll, provided λ remained unchanged 1·50 at 5·15 mm depth, as was found above. With the brass 2·5 mm off the oil, in fact twelve rings with a rather large centre ring could be observed, when the lid was lifted finally. The result was the same at 1·5 mm distance. Yet at 0·5 mm distance there were thirteen rings. Surprisingly the roll at the rim turned now inwards at the surface, while in all other experiments it turned outwards. This is probably due to the air pocket between the oil and the brass, where the brass avoids the raised oil level at the rim. This shows how sensitive the direction of motion is against small changes at the rim. Consequently the innermost thirteenth ring turned downward at the centre. No single cell can therefore be formed; there were rather four cells in the innermost ring and then 10–16–22–28–31 cells in the other rings and one undisturbed roll at the rim. Four cells at the centre call for a larger than harmonic centre ring but give, as has been observed with even numbers of rings under a glass plate, the most regular pattern. To summarize, it has been found that the number of rings can be increased from twelve to thirteen by bringing the lid closer to the oil surface, in other words λ has been reduced by varying q_0 at constant d.

Finally, the results of these experiments have been checked with a silicone oil of 10 stokes viscosity. The onset of convection and the establishment of the hexagonal pattern was exactly of the same type as described above. The wavelength varied a little, eleven harmonic rings appeared at 6·44 mm depth, corresponding to $\lambda = 1{\cdot}41$. This change might be caused by a variation of q_0 or be due to a 3 % increase in the thermal conductivity.

5. Supercritical motions

Supercritical motions are of major interest, since the non-linear terms in the equations for the convective motions become essential. Up to Rayleigh numbers of around three times R_c, a hexagonal pattern can remain virtually unchanged, as shown in figure 6. Yet if the original pattern had been less regular, the number of cells can increase by as much as 10 %. It appeared as if regular hexagons, once established, never changed very much, either naturally or by artificial stabilization through the aluminum powder settled at the cell centres. A systematic increase or decrease of the wavelength of the motions would have meant a decrease or increase in the number of the cells and thus be easily observable. But once a regular pattern had established, the fluid could apparently not make up its mind which cells had to disappear or where to create new ones. To circumvent this difficulty it was tried to produce the hexagonal pattern on a supercritical ring pattern. This could only be done if the applied temperature difference varied rapidly; under stationary conditions no supercritical ring pattern has ever been observed under an air surface. It was easy and reproducible to establish fourteen supercritical rings in a 5·15 mm thick oil layer, if the lid was cooled down in about 10 min to increase the overall temperature difference from around 180 to 440 μV. At about that temperature difference, a symmetric fourteenth centre ring appeared, followed by a very rapid transformation to a rather regular hexagonal pattern with four cells in the centre. Increasing the temperature difference further did not change the pattern substantially. At a still faster increase of the vertical temperature difference even sixteen rings could be observed, although the innermost rings were no longer regular and the hexagonal pattern likewise. These observations indicate that the wavelength decreases with supercritical heating, although this should be considered with great caution, since we have applied a time-dependent technique. A decrease of the wavelength would be in contrast to the 30 % increase of the wavelength at $5R_c$ under a rigid lid, observed by Koschmieder (1966). To summarize, supercritical motions did not give unique results, it can only be stated definitely that the number of cells at supercritical ΔT never decreased.

6. Conclusions

It has apparently never been observed before that a hexagonal cell pattern under an air surface developed on a pattern of rolls. This is probably due to the fact that other experiments have not sufficiently cared for a proper definition of the conditions at the upper surface. As the rolls develop they have the proper wavelength and they need the critical temperature difference to cover the whole plane. The circular rolls reflect the unavoidable existence of a lateral wall. The wall here is thin and surrounded by an excellent thermal insulation, to reduce the non-uniformity of the temperature field in the fluid near the rim. As already mentioned, experiments by Koschmieder (1966) indicate that the walls determine the type of convective motions on a uniformly heated plane. The appearance of the concentric rolls here only confirms this observation, under conditions which are, due to the air surface, much more favourable for the fluid to select any type of motion. The importance of the lateral walls has, furthermore, been confirmed by a recent theoretical study of Davis (1967) about convection in rectangular boxes. It might be argued that the rings are the consequence of a radial temperature dependence and thus are not related to the Bénard problem. But a substantial radial temperature difference would produce concentric rings with alternatively larger and smaller horizontal sections, as was shown by Koschmieder (1967). This phenomenon is clearly visible in the first picture shown by Segel. Since the rings here appeared consecutively and had all the same sections, a substantial radial temperature gradient is very unlikely. The question remains, whether the hexagons should not appear at random in the resting fluid. They do, as long as one does not care for the conditions on top of the fluid. As soon as one defines the temperature on top of the fluid, which is physically sound, the rings appear, and they appear more regularly, the thinner the air layer is. As soon as all rolls have developed they break down into hexagons, apparently due to almost infinitesimal disturbances, at least they are too small to disturb the rolls. The conversion of rolls into hexagons seems thus to be caused by a real instability. It looks as though Nield's theory finally describes the essential features of the onset of convection under an air surface. As a linear theory, his theory does not determine the form of the motions. Therefore the question of why and how the transformation into hexagonal cells takes place, remains open. It is remarkable that such a spectacular phenomenon has not yet been explained, 67 years after it became known. A first attempt for an explanation is Segel & Scanlon's (1967) paper.

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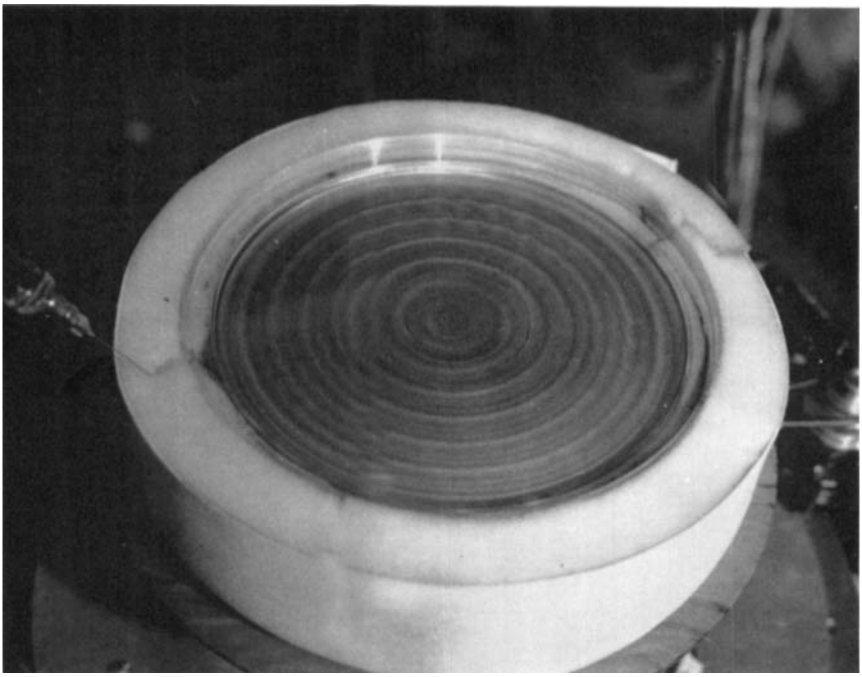

FIGURE 1. Concentric ring pattern, $\Delta T = 380\ \mu V$, $t = 0$.

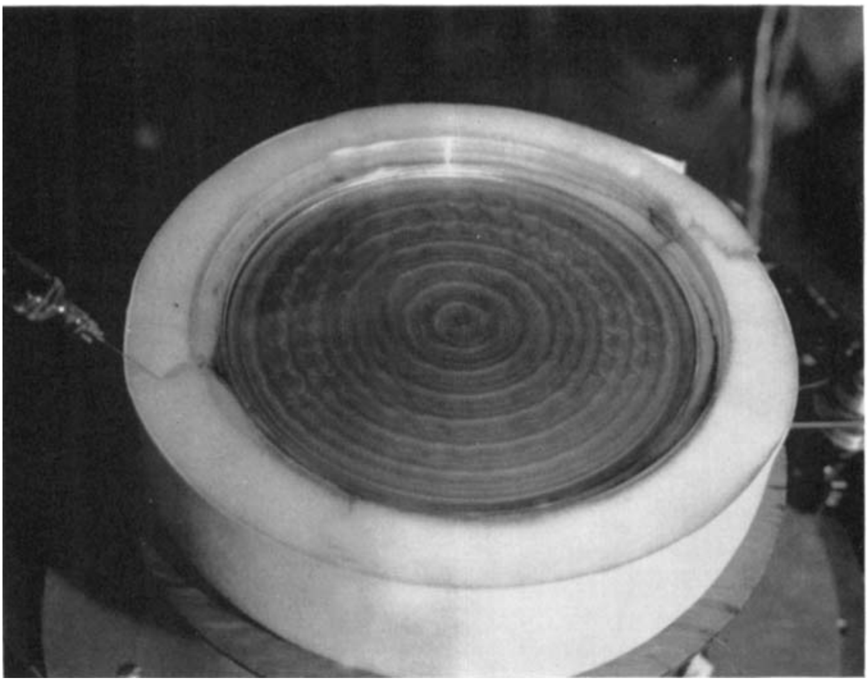

FIGURE 2. Breakdown of axial symmetry, $\Delta T = 390\ \mu V$, $t = 8$ min.

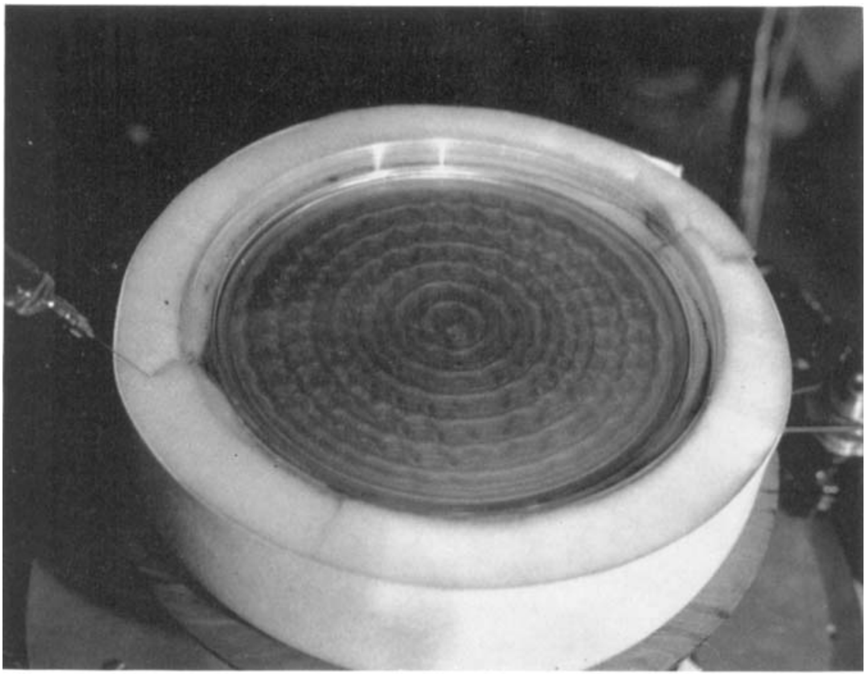

FIGURE 3. Breakdown of axial symmetry, $\Delta T = 400\ \mu V$, $t = 15$ min.

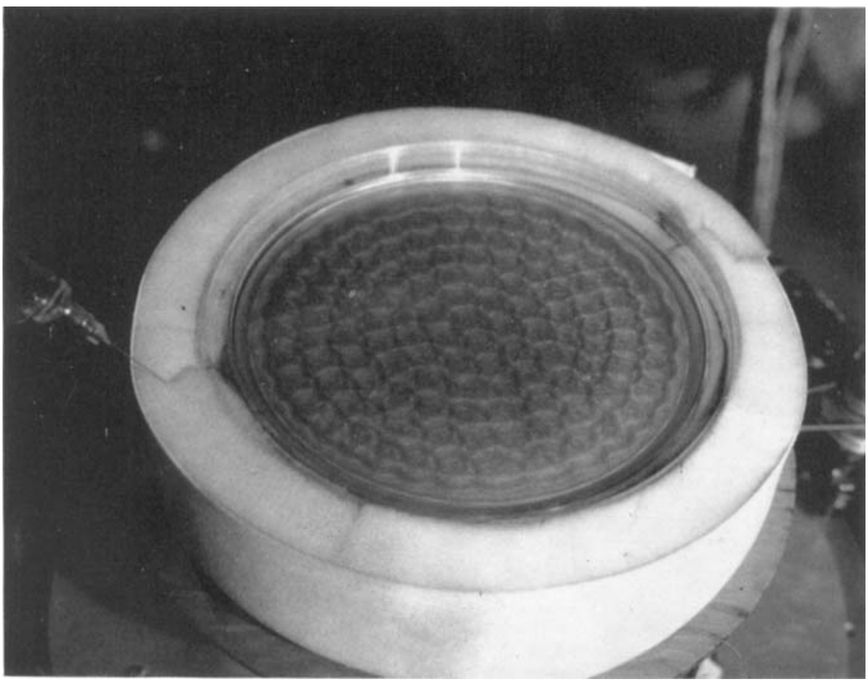

FIGURE 4. Hexagonal pattern, $\Delta T = 400\ \mu V$, $t = 35$ min.

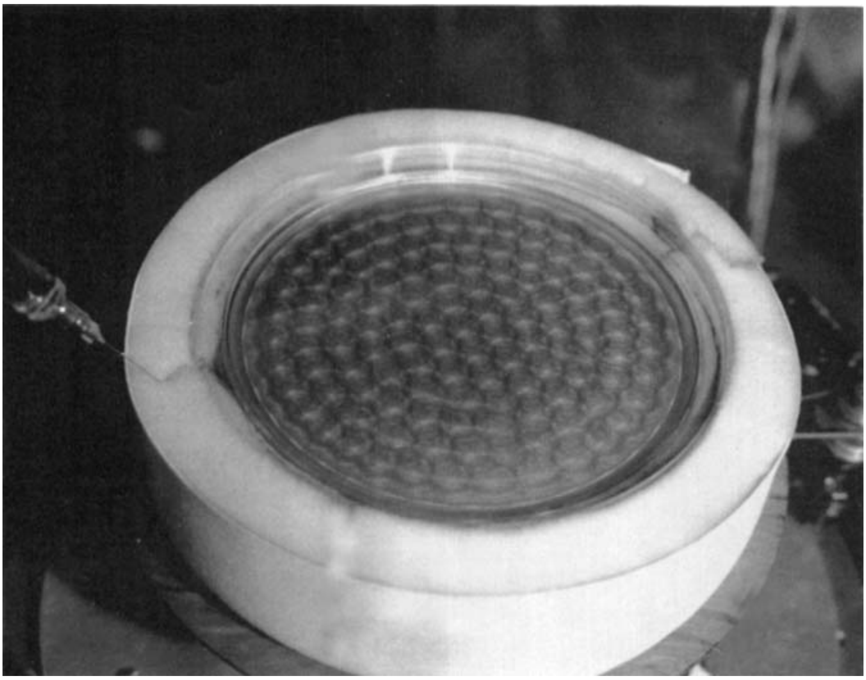

FIGURE 5. Hexagonal pattern, $\Delta T = 400\,\mu$V, $t = 85$ min.

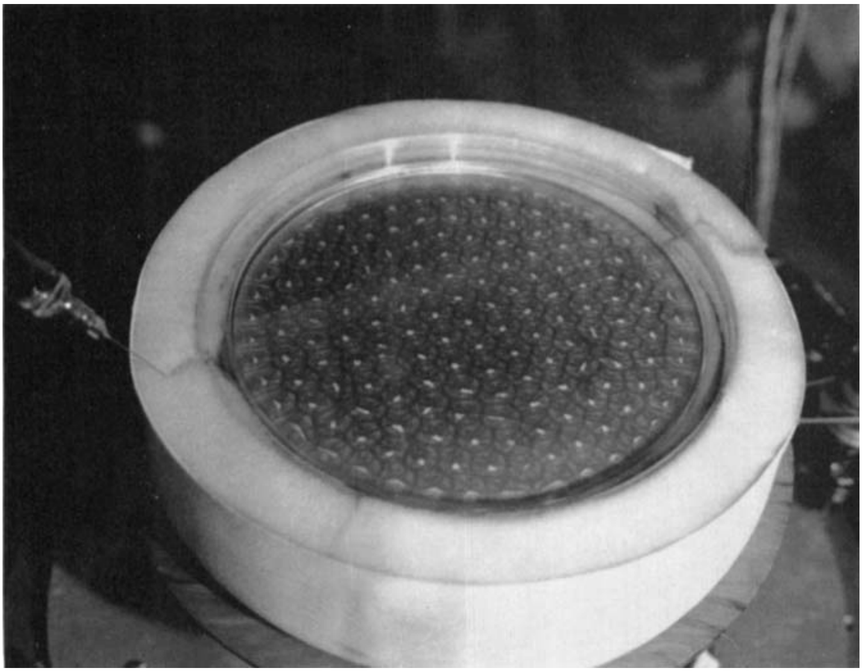

FIGURE 6. Supercritical hexagonal pattern, $\Delta T = 960\,\mu$V, $t = 315$ min.